On the Motion of a Rigid Body with a Liquid-Filled Cavity:

A Mathematical Analysis

Objective of this course is to provide a comprehensive analysis of the motion of the coupled system constituted by a rigid body containing an interior cavity that is entirely filled with a liquid. The main goal is to show how the presence of the liquid may dramatically influence the dynamics of the body. For this reason, we shall first analyze the case when the cavity is empty and review classical results about the motion of a free or constrained rigid body, moving under the action of a given system of forces. Successively, we will first consider the situation of a cavity filled with an inviscid liquid and show, already in the case of potential flow, how the stability properties of the rigid body may be affected by the liquid, to the point that motions that are unstable in absence of liquid become now stable and vice-versa. The last, and more substantial part, will be dedicated to the case when the liquid is viscous and governed by the Navier-Stokes equations. We will prove that then the liquid has always a stabilizing effect on the motion of the body and that the terminal state (as time goes to infinity) of the coupled system must be a permanent rotation where the liquid is “frozen” inside the cavity and the coupled system moves as a single rigid body.